

Nombre: Par: UEMTN 2015-2016

1. Sea la función $f(x) = 3x$, y $g(x) = 2x - 5$. Se define la función h como $h(x) = (f \circ g)(x)$. Encuentre la regla de correspondencia de:

(a) $h(x)$.

(3)

(b) (i) $h(7)$

(1)

(ii) $h^{-1}(-3)$

(3)

(Total 6 marks)

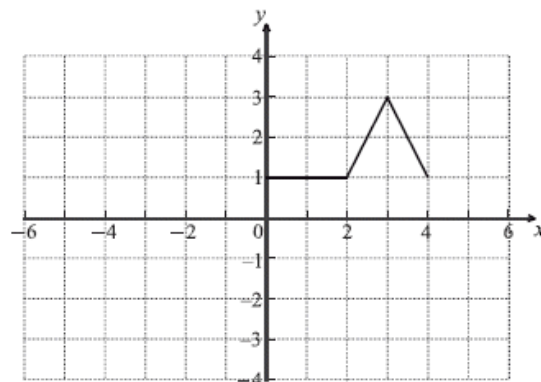
2. Sean las funciones $f(x) = x^2 + 6x - 1$ y $g(x) = 6 - x^2$, calcule los valores de x para los cuales se cumple que:

(a) $f(x) + g(x) = 0$

(b) $f(x) - g(x) = 0$

(Total 6 marks)

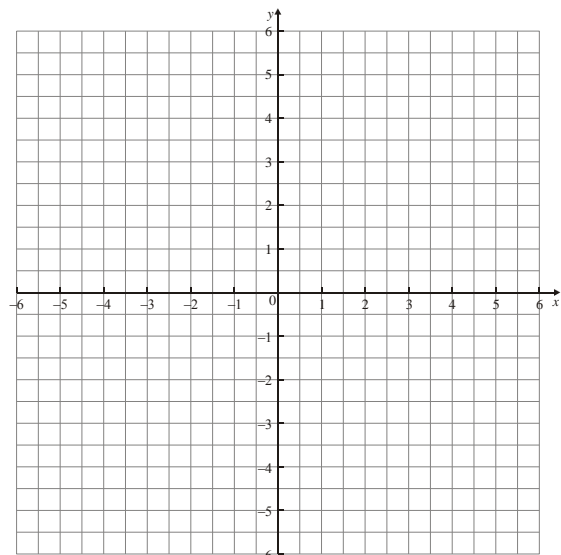
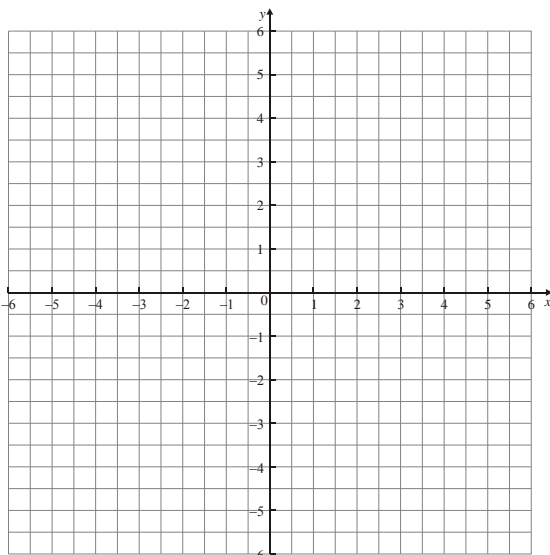
3. Para la gráfica de la función que se muestra a continuación:



Grafique:

(a) $y = -f(x+2) - 3$

(b) $y = \frac{1}{2}f(-x) + 2$



4. Para la siguiente función $f(x) = \frac{2-3x}{x-3}$ para $x \neq 3$:

(a) Determine $f^{-1}(0)$

(3)

(b) Determine la regla de correspondencia de $f^{-1}(x)$

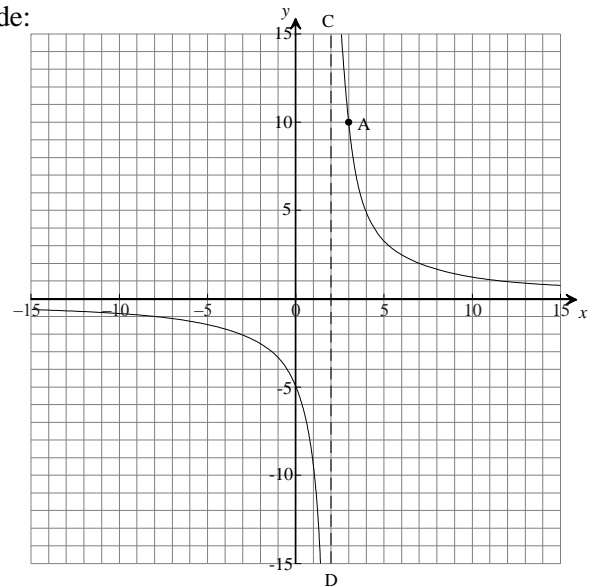
(5)

5. De acuerdo con la gráfica de $y = g(x)$, escriba los valores de:

(a) $g(4) + g(0)$

(b) $g(-8) + g^{-1}(5)$

(c) $g^{-1}(10) - g^{-1}(1)$



6. Sea la ecuación cuadrática $2x - 3\sqrt{x} - 2 = 0$. Si se tiene que $u = \sqrt{x}$:

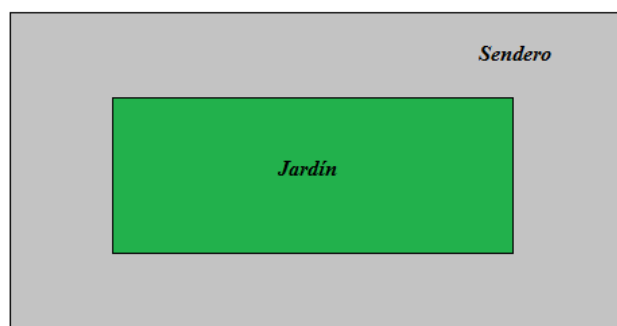
(a) Escriba la nueva ecuación cuadrática que se forma en términos de u . (1)

(b) Exprese la ecuación anterior en su forma factorizada. (2)

(c) Determine los valores de u que satisfacen la ecuación anterior. (2)

(d) De lo anterior, escriba los valores de x que satisfacen la ecuación $2x - 3\sqrt{x} - 2 = 0$. (2)

7. Se tiene un terreno rectangular de lados 40 m de largo por 30 m de ancho. Se desea construir un jardín rectangular de tal manera que se encuentre en el centro del terreno y que tenga un sendero de x metros de longitud en su perímetro. El área del jardín debe ser el 60% del área total del terreno. Determine el valor de x .



8. Determine los posibles valores de k para que la ecuación $(2k+1)x^2 + 3kx - 2k + 1 = 0$ no tenga soluciones reales.

9. The quadratic function $f(x) = p + qx - x^2$ has a maximum value of 5 when $x = 3$.

(a) Find the value of p and the value of q .

(4)

(b) The graph of $f(x)$ is translated 3 units in the positive direction parallel to the x -axis. Determine the equation of the new graph.

(2)

(Total 6 marks)

10. (a) Express the quadratic $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where $a, b, c \in \mathbb{Z}$.

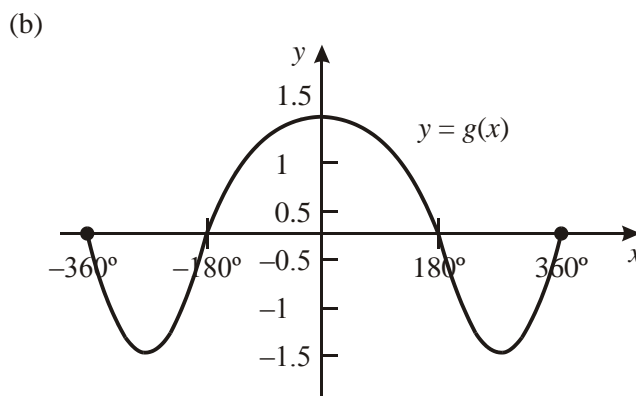
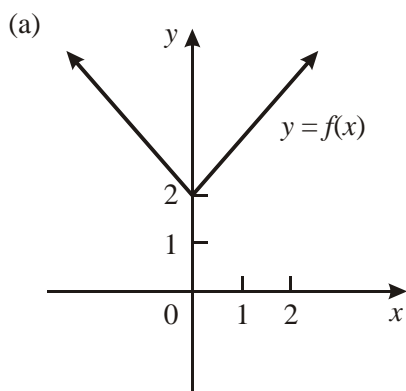
(3)

(b) Describe a sequence of transformations that transforms the graph of $y = x^2$ to the graph of $y = 3x^2 - 6x + 5$.

(3)

(Total 6 marks)

11. The diagrams below show the graphs of two functions, $y = f(x)$, and $y = g(x)$.



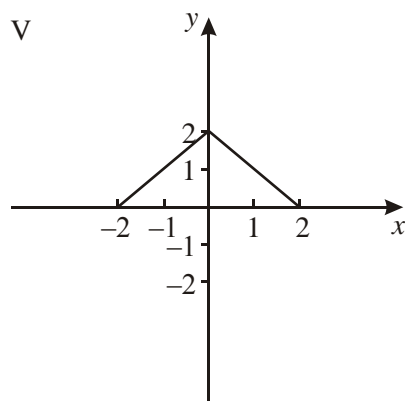
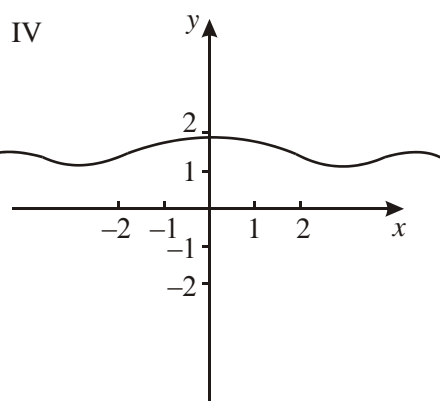
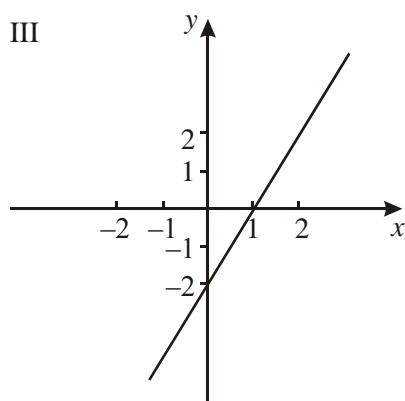
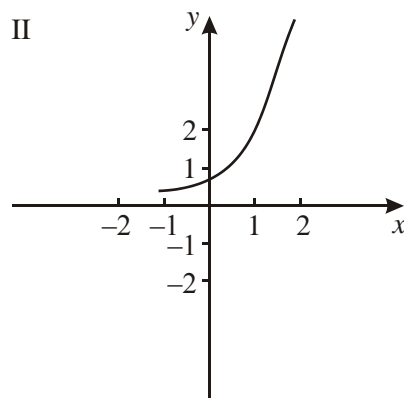
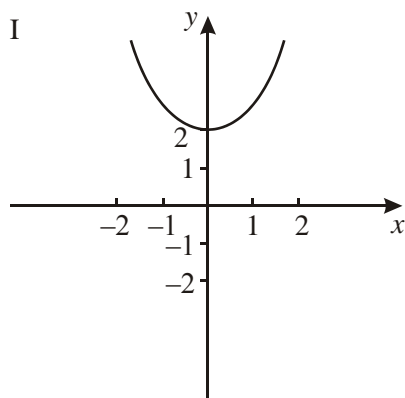
State the domain and range of

(a) the function f ;

(b) the function g .

(Total 8 marks)

12. The following diagrams show the graphs of five functions.



Each of the following sets represents the range of one of the functions of the graphs.

- (a) $\{y \mid y \in \mathbb{R}\}$
- (b) $\{y \mid y \geq 2\}$
- (c) $\{y \mid y > 0\}$
- (d) $\{y \mid 1 \leq y \leq 2\}$

Write down which diagram is linked to each set.

(Total 4 marks)

13. Let $f(x) = \sqrt{x+4}$, $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.

- (a) Find $(g \circ f)(3)$.
- (b) Find $f^{-1}(x)$.
- (c) Write down the domain of f^{-1} .

(Total 6 marks)

14. Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

- (a) Find $h(x)$.
- (b) Find $h^{-1}(x)$.

(2)

(3)

(Total 5 marks)

15. Consider the functions $f: x \mapsto 4(x - 1)$ and $g: x \mapsto \frac{6-x}{2}$.

- (a) Find g^{-1} .
- (b) Solve the equation $(f \circ g^{-1})(x) = 4$.

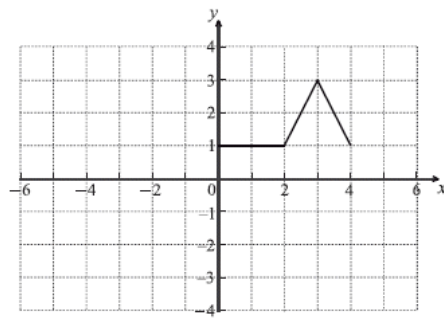
Working:

Answers:

- (a)
- (b)

(Total 6 marks)

16. Consider the graph of f shown below.



- (a) On the **same** grid sketch the graph of $y = f(-x)$.

The following four diagrams show **images** of f under different transformations.

Diagram A

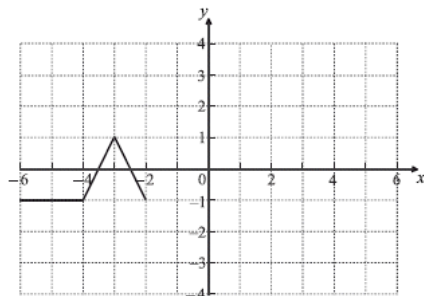


Diagram B

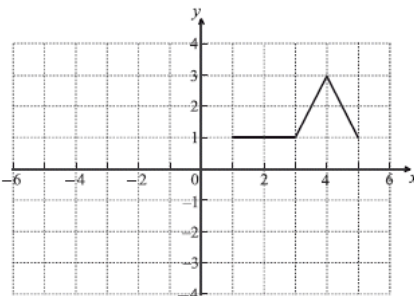


Diagram C

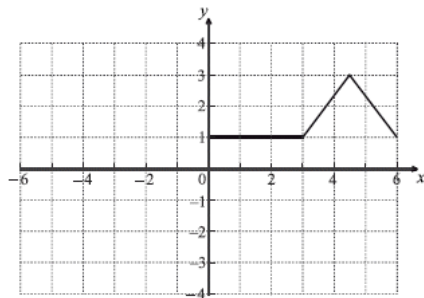
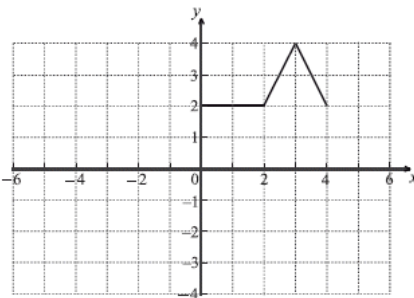


Diagram D



- (b) Complete the following table.

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	
Maps f to $f(x) + 1$	

(2)

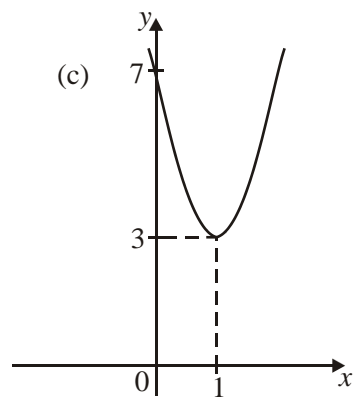
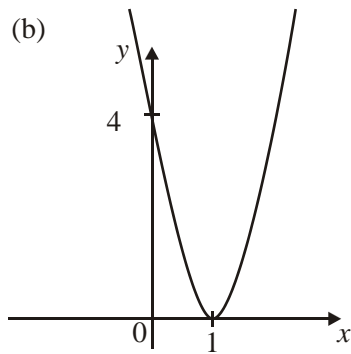
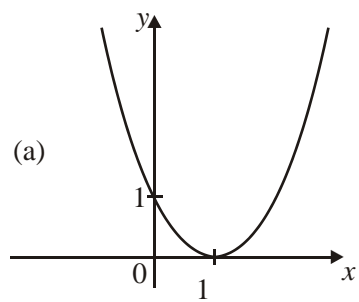
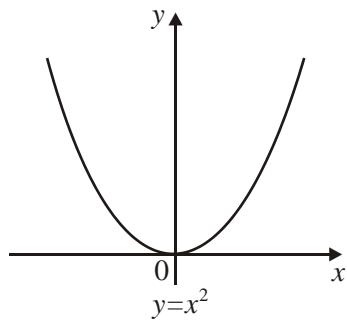
- (c) Give a full geometric description of the transformation that gives the image in Diagram A.

(2)

(Total 6 marks)

17. The diagrams show how the graph of $y = x^2$ is transformed to the graph of $y = f(x)$ in three steps.

For each diagram give the equation of the curve.



Working:

Answers:

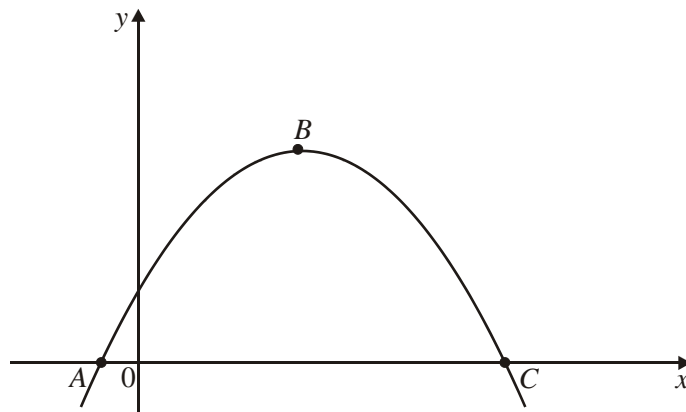
(a)

(b)

(c)

(Total 4 marks)

18. The diagram shows the parabola $y = (7 - x)(1 + x)$. The points A and C are the x -intercepts and the point B is the maximum point.



Find the coordinates of A , B and C .

Working:

Answer:

.....

(Total 4 marks)

19. Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots.

(a) Find the value of k .

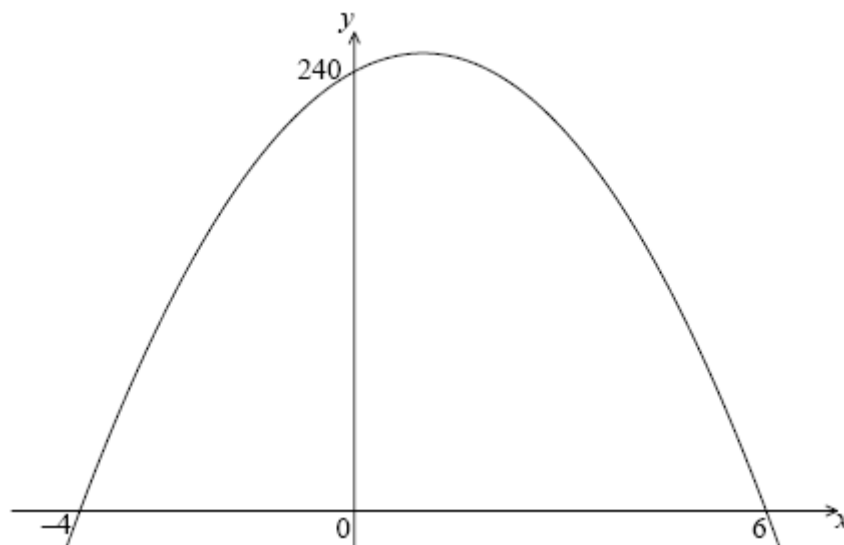
(5)

(b) The line $y = p$ intersects the graph of f . Find all possible values of p .

(2)

(Total 7 marks)

20. The following diagram shows part of the graph of a quadratic function f .



The x -intercepts are at $(-4, 0)$ and $(6, 0)$ and the y -intercept is at $(0, 240)$.

- (a) Write down $f(x)$ in the form $f(x) = -10(x - p)(x - q)$. (2)
- (b) Find another expression for $f(x)$ in the form $f(x) = -10(x - h)^2 + k$. (4)
- (c) Show that $f(x)$ can also be written in the form $f(x) = 240 + 20x - 10x^2$. (2)

A particle moves along a straight line so that its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v = 240 + 20t - 10t^2$, for $0 \leq t \leq 6$.

- (d) Find the value of t when the speed of the particle is greatest.